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# Exploring Mispricing and Arbitrage Strategies in the U.S. Stock Market: A Comprehensive Analysis Using CBOE Options Data

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## Abstract:

This research examines the suitability of options pricing in comparison to the prices of their underlying stocks. The analysis employs real-world options data from the Chicago Board Options Exchange (CBOE) market, covering the period from mid-January 2017 to the end of the month. The accuracy of options prices is assessed using multiple criteria: upper and lower bounds, strict lower bounds, exercise price, time to maturity, convexity conditions, put-call parity conditions, and the Black-Scholes model. These criteria aid in identifying optimal trading positions and evaluating various profit factors. The findings indicate significant arbitrage opportunities when the put-call parity condition is violated. Additionally, the study investigates the factors that contribute to these pricing discrepancies.

## Keywords:

Options Price; Arbitrage Opportunity.

## 1. Introduction

The illiquidity of the options markets results in temporary deviations of options prices from their fundamental values. Being able to find arbitrage opportunities gives investors a chance to exploit the money machine without actually taking any risks theoretically. A successful options trading which is priced properly consists of a holistic establishment of upper and lower bound, convexity conditions, put-call parity conditions, and Black-Scholes model. In this paper, we will examine each factor holistically, including any elements that might contribute to the fluctuation. The following work will show how to use real-world data to come up with the results; then other underlying elements that would help support the results will be discussed and exploited.

## 2. Literature Reviews

In the paper Index Options Prices and Stock Market Momentum, Kaushik Amin, Joshua D. Coval, and H. Nejat Seyhun innovatively checked the probability of boundary violations and used the $\chi^2$ test, suggesting that boundary condition violations happen more likely when the stock prices change and are somewhat more significant for the precast period by using the data of OEXoptions from 1983 to 1995 [1]. Researchers used the iterate model, where $C(\sigma_i)$ represents the price of the given potion with an implied volatility of $\sigma_i$ computed from the binomial model; $C(\sigma^*)$ represents the observed options price; and Vega represents the partial derivative of the binomial options price with respect to $\sigma_i$ [1]. The result suggested the total implied that declining stock prices would lead to increases in volatility. Furthermore, after examining the influence of past stock returns on the volatility smiles (the estimating function of the exercise price with U-shaped implied volatility), they found the market momentum effect consistently declines with increasing moneyness but cannot explain the volatility smiles. Hence, writers defined a new measure of volatility smile and examined the relation in between the past stock returns and the present stock returns. After that, overall volatility spread become the concern, and this differential response was precisely quantified.

Result shows that deep-in-the-money options and deep-out-of-money have the highest price pressure, while at-the-money options appears to have the lowest price pressure. To further research the potential sources of price pressure, writers analyzed the kurtosis and skewness, finding that past returns appears not to be a proxy variable for higher moments of stock return, and the market momentum hypothesis is not rejected even other factors are controlled and market momentum affects options valuations is not produced by worries [1].

In "volatility uncertainty and the cross-section of options returns," published by Jie Cao, Aurelio Vasquez, Xiao Xiao and Xintong Zhan, a holistic examination of the relation between the future delta-hedged equity options returns and uncertainty of volatility was finished [2]. Results showed that when the volatility is uncertain, the delta-hedged options returns would decrease consistently. To test the hypothesis, the authors constructed a delta-hedged options portfolio where volatility changes are the main factor in the portfolio returns. To measure the volatility of the stock, three different methods were used: Firstly, volatility is estimated from an EGARCH model in the timespan of 252 trading days. Secondly, implied volatility from 30 days to maturity options. Thirdly, intraday realized volatility in the span of 5-minute stock. The authors found that these three volatilities of volatility (VOV) measures could predict returns from future options. To further explain where VOV predictability comes from, the authors explained that days around earning announcements do not affect return spread [2]. Second, rather than by any systematic factors, most of the predictability is driven by VOV, while it is more costly for market makers to hedge options with the high volatility of idiosyncratic volatility. Third, the authors decomposed VOV into VOV+ and VOV-, founding that options sellers prefer VOV+ to VOV-. In the end, the authors explain that when volatility is costly to forecast and options sellers tend to charge a higher premium for equity options [2]. To further illustrate the correlation between idiosyncratic volatility of options on stocks and profit returns, Jie Cao and Bing Han assume that on average, options on low idiosyncratic volatility stocks earn significantly higher returns than options on high idiosyncratic volatility stocks [3]. To approve this hypothesis, writers examined a cross-section of options on individual stocks for every month. It is interesting to see that for a more meaningful result, before the examination, (the author did) several variables were controls which is unexpected. For instance, the author did control for volatility-related options mispricing, finding that Goyal and Saretto (2009) applied wrong data to estimate stock price [4], but they figure out using cross-sectional distribution will limit the volatility of mispricing [3]. However, our paper is more focused on the frequency

of mispricing and how to apply strategy to future market, Cao, J, and Han, B (2013) didn't address this question, instead they were trying to figure out the how the change of idiosyncratic volatility of options effect profit return [3]. Their study goal is more prone to analyze how the market will affect the volatility of options on stocks, but this paper is more prone to analyze how individual investor's behavior causes the frequency of mispricing. Even though, they stated useful information for our study is that volatility risk premium or volatility-related options mispricing are not necessary factors of negative cross-sectional relation between delta-hedged options return and idiosyncratic volatility of the underlying stock [3].

However, will volatility be mispriced? Goyal and Saretto (2007) found an economically important source of mispricing in the options implied volatilities [4]. It is the trader's conviction that lies behind a volatility trade, i.e., the market expectation about future volatility that are implied by the options price is not entirely correct. Because at least an estimate of the parameters that characterize the probability distribution of future volatility is necessary for all the options pricing models, and volatility mismeasurement is indeed the most obvious source of future options mispricing. Underlying stocks are sorted according to the difference between historical realized volatility and at-the-money implied volatility. Then portfolios are constructed using straddles and delta-hedged calls and puts. The authors suggested that a zero-cost trading strategy which involves a long position in the portfolio with a huge positive difference in this historical realized volatility and implied volatility (and vice versa), results in significant average monthly returns, both economically and statistically [4]. The results hold regardless of market conditions, amount of risk attached to the stock, industry groupings, or liquidity of options. Besides, linear factor models do not explain the results. In this paper, Goyal and Saretto (2007) empirically investigated this mispricing conjecture that comparing a measure of historical realized volatility to the market volatility forecast is a possible way to identify whether an option is mispriced or not, where the key aspects are implied by the optionsprice by researching the cross-section of equity options in U.S. market [4].

In Coval and Shumway's paper "Expected Options Returns", authors mainly focused on examining options returns through different asset pricing theories and backgrounds [5]. The first important thing to understand is the risk-return characteristics of options returns. The authors considered two main ideas: leverage effects and curvature of options payoffs. In particular, using Black-Scholes Model, they analyzed that call options should earn expected returns which exceed those of the underlying security, while put options should earn expected returns below that of the underlying security, and they carefully tested and rejected the hypothesis of "delta-neutral options positions should earn the risk-free rate on average" [5]. They built the test mainly to directly examine the leverage effect on various models so that they can use economic terms to check the violations of market efficiency, and therefore consider the economic payoff for taking particular risks. Experiments indicate that options return largely conform to most asset pricing implications, but due to systematic risks, both call and put contracts earn exceedingly low returns. However, they also figured out a strategy of buying zero-beta straddles that have an average return of around -3 percent per week, showing that pricing assets may significantly cause by systematic stochastic volatility [5]. They also came upwith a possible extension of their results: investigations of whether volatility risk is priced in the returns of other assets.

## 3. Data and Methodology

In this section, we are going to introduce different methods of testing for violations. This includes more details about principles, underlying assumptions, and potential limitations. The notations we use: *C(c)* denotes as the call price, *P(p)* denotes as the put price, *S* denotes as stockprice, *X* denotes as the strike price, respectively.

### 3.1. Put-Call Parity

Before the checkout of the Black-Scholes model, firstly we pay attention to the upper bound, lower bound, and Put-Call Parity conditions, and it is necessary to introduce the principles of them. In the Put-Call Parity, the following conclusion has been obtained (considering the American options, we will not discuss the present value of exercise prices):

Once the equation is impossible, we can do arbitrage. In reality, it is possible to find the inequation of the two sides. Under this condition, we can buy low and sell high. Here we just introduce and demonstrate the violation of upper bound. For instance, for call options, if $c>S$, we can do arbitrage by selling the calls and buying the stocks, and not loss. (Also, if $c=S$, we can do the same operation), then the “upper bound” on call price displays, i.e., it cannot exceed stockprice *S*.

Nevertheless, American Put-Call Parity is different from European one. More specifically, we write down the equation of American Put-Call Parity without dividend:

where $S_0$, $X$, $C$, $P$, $r$, $T$ denote current stock price, exercise price, call price, put price, risk-free rate and maturity, respectively. Usage of natural constant $e$ means compounding continuously. Similarly, if the inequations are violated, we can arbitrage.

### 3.2. Upper Bound and Lower bound

Base on the data from CBOE market ranging between half of a month to a full month in January, 2017, this section will discuss below how to find the risk-free arbitrage opportunity and the method used for calculating the percentage of a risk-free arbitrage opportunity. First, for the lower bounds on call prices, in general, call options must sell above their call intrinsic values atall time C>S-X (C stand for call price, S stand for the stock price and X stand for exercise price), if the call intrinsic values are higher than call price it is a money machine. After minus stock prices and exercise prices of call options we compare it to call prices to see which one is higher. Second, upper bounds on call prices must be that stock prices always higher than call prices C<S, if not there is a risk-free arbitrage opportunity. Also, call prices and stock prices have been compared, Third, lower bound on put prices must always sell above its intrinsic value P>X-S, if its intrinsic value is higher than put prices, it creates a money machine. Then the intrinsic value of the put options is calculated, and compared to put options prices to find out which prices are higher. Finally, upper bounds on put prices must always be exercised higher than put prices, i.e., P<X, if not, it creates a risk-free arbitrage opportunity. Hence, all the put price are selected and compared to the exercise price to find out which prices are higher.

### 3.3. Convexity condition

To be convex, succeeding equal increases in exercise prices must decrease the call prices by less and less. And a $5 increase in exercise price decreases the call prices by less than $5, but in decreasing increments. Succeeding equal increases in exercise prices must increase the put prices by more and more. And a $5 increase in exercise price increases the put prices by less than $5 but in increasing increments. If the convex condition is violated, you can do a butterfly or vertical spread to arbitrage, which means buy low and high and sell 2 middles.

### 3.4. Black-Scholes Model and Delta

The pricing of options depends on the model (i.e., the prices obtained by different models are different), so the arbitrage strategy cannot be implemented without the framework of the model. Only if the market meets the framework of this model can the arbitrage strategy be implemented? The basic assumptions of the Black - Scholes model is the following:

(1) During the life of the options, there is no dividend payment and other distributions of the underlying asset of the options (to simplify, the asset here denotes stock);

(2) When dealing the options or the stocks, transaction cost can be neglected;

(3) The short-term risk-free interest rate is known and remains unchanged over the lifecycle;

(4) Any purchaser of securities can borrow any amount of money at a short-term risk-free rate;

**(5)** Short selling is allowed, and the short seller will receive the money for the price of the stock sold short on that day immediately;The options are European;(The American options in the data are temporarily treated as European options)

(6) All securities trading occurs continuously, and short-term stock price random walk.

(7) Stock prices are subject to a lognormal distribution.

The Black-Scholes formula applied to value a European option written on a stock:

where $d_1$ and $d_2$ are preliminary calculations. $N(x)$ represents the cumulative probability function for a standardized normal variable, which is also the probability that a variable with a standard normal distribution will be less than x. $N(d_1)$ and $N(d_2)$ are to assess the probability that the stock price will exceed the strike price so that the call options will end up being exercised at maturity. Specifically, $N(d_1)$ measures the probability that the present value of future stock price will exceed the current stock price; $N(d_2)$ measures the risk-adjusted probability that the call options will be exercised.

Delta is defined as the rate of change of the options price concerning the price of the underlying asset. The diagram below shows the relationship between a call price and the underlying stock price:

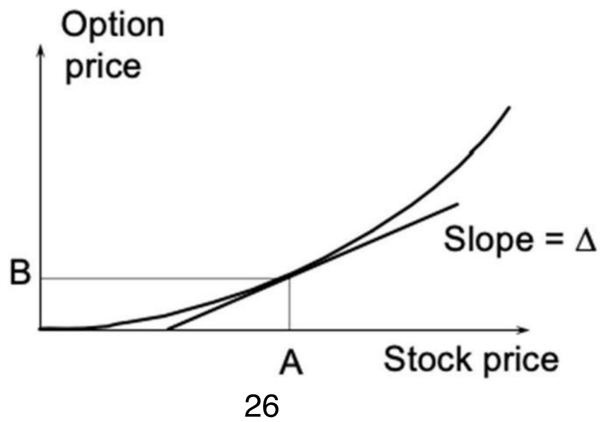

Figure 1. Relationship between call price and underlying stock price

The Black-Scholes Options Pricing Model can be used to calculate Delta. The Delta of a long European call options on a non-dividend-paying stock is $N(d_1)$. The Delta of a long European put options on a non-dividend-paying stock is $N(d_1) - 1$.

For a call option, if $C > BS$ it is expensive, ignoring the transaction fee, you need to sell the actualcall and buy a synthetic call or buy $N(d_1)$ shares. If $C < BS$, it is cheap, you need to buy the actualcall and sell synthetic call or sell $N(d_1)$ shares.

For a put option, if $P > BS$ it is expensive, you need to sell the actual put and sell $N(d_1)$ -1 of the stock. If $P < BS$ it is cheap, you need to buy the actual put and buy $N(d_1)$ -1 of the stock.

## 4. Findings

We then implemented our testing methods into our data and came out with our arbitrage strategies and conclusions.

### 4.1. Upper Bound Violations

First, the actual data are checked. Before calculating, we control the maturity to ensure the consistency, and group each call/put price and stock price in the CBOE market (from 1.1 to 1.31) in 2017, getting 1048575 groups for each side of calls and puts. 3057 groups of call options and 2758 of put options offer the arbitrage opportunities based on the analysis of upper bound, and the proportions are 0.29% and 0.26%. The average scale of profit in upper bound arbitrage is about \$1.31 per share of put and \$1.08 per share of call. To arbitrage, take call options for an example, we can sell the calls and buy the corresponding stocks, then it will be a money machine.

More interesting finding is not about the data which violate the upper bound, but the factors that affect the violated conditions. Because the maturities have been adjusted to be the same, we will start with the stock prices. More precisely, we will figure out how much will the profits be influenced when stock prices change under the violated condition, and the result is, there is a significant negative relationship between them.

Here is the figure:

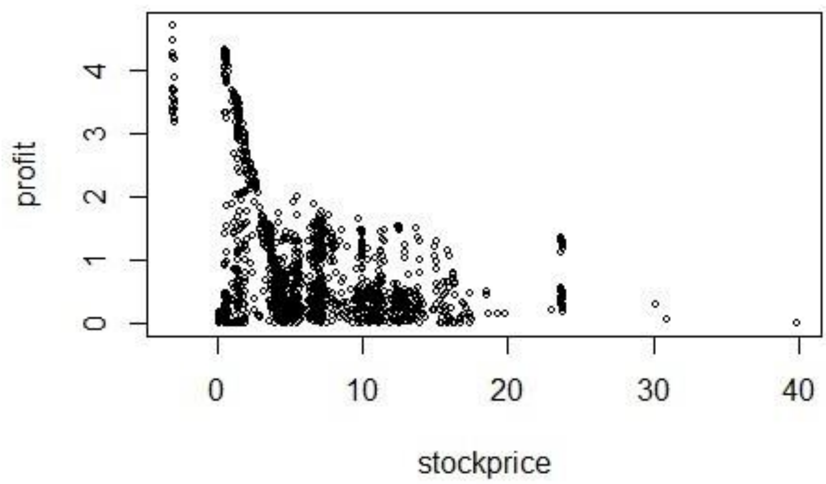


Figure 2. Relationship between stock price and profit

To qualify the effect, we add the trendline, considering that the linear fitting is too inexact whenthe stock price is large enough (the profit will be too negative), we use a logarithmic function to model the relation. Indeed, the result is better than linear fitting, and it is as follows:

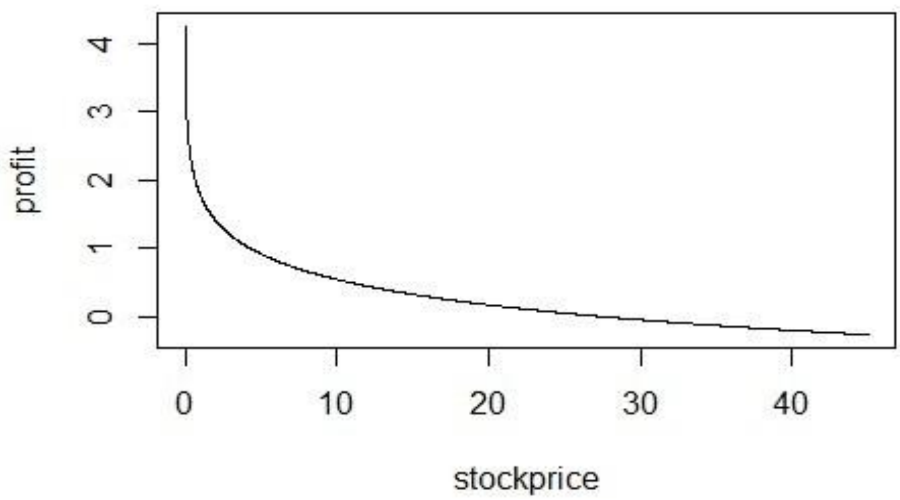


Figure 3. The adjusted fitting-figures (based on Logarithmic fitting)

By constructing a graph of a function, the effect brought by stock price displays. We can conclude when the stock price is low (or, lower than \$5), the negative effect of stock prices is strong because of the large slope, when stock prices get higher and higher, the strength of the effect decreases, but still negative.

After checking the upper bound, we turn to Put-Call Parity. Simply put, we will check if the inequations of American options make sense. (We use LIBOR to measure the risk-free rate). Surprisingly, there are too many violated data. Before discussing the reasons why these data violate the Put-Call Parity, we firstly show the data in detail.

After checking the left side of the inequation, we find 488404 violated data in all of 1048575 data, and the violated proportion reaches 46.58%, a very high scale. Meanwhile, the average scale of profit is about \$0.88 per share. Data which violate the right side of the inequation show a number of 473827, a proportion of 45.19% and an average profit scale of \$0.82 per share.

Apparently, the violated data of Put-Call Parity are far more than the ones of upper bound and lower bound. In theory, it is impossible to violate the Put-Call Parity so many times, so the strange phenomenon needs an explanation. For the assignment of each variable, there is a flaw that we use the LIBOR whose maturity is between 2 weeks to 1 month to denote the fixed risk-free rate, while in reality the maturity of each options can be different. However, this impact just has an influence on the violation of right side of the inequation, making the result less precise. For the left side, the assignment of risk-free rate cannot bring any impact, so there must be another explanation. Besides the options premium, fees may be a reason causing the huge proportion of violated data. Because of the fees, investors have to give up the arbitrage opportunities, or we can say there are no arbitrage opportunities when considering the fees. Hence, to get the “real” violated data, we should consider the payment of fees.

## 4.2. Lower Bound Violations

To exploit profit opportunity, we also need to take a close look at any violations of lower bound or strict lower bound. For call options, in general, C must be greater than S-X, and for the strict lower bound, C must be greater than S-PV(X). Based on 1048575 data we collected, we found 469, 0.04% in total, put options and 713, 0.068% in total, call options violate the lower bound rule, which provides investors an opportunity to make arbitrages. The average scale of profit in lower bound arbitrage is about \$0.31 per share of put and \$0.082 per share of call.

If we assume such an event is not happening under systematic conditions; in other words, there will be a money machine: risk-free arbitrage opportunity. For call options, we can buy low, sell high, and make money for sure. To best illustrate this, we take a violation on lower bound for example: if we assume C=11, S=$40, X=$40, Rf=10%, T=1 year; then we would buy the call for $11, short the stock for $50, and lend the present value of the exercise price,

$36.3 (36.36=40/(1+R)). We could get an immediate $2.64 (50-36.36-11) in cash. For put options, if the put is selling below intrinsic value; then it will create a money machine. We assume lower bound is violated, and P <PV(X)-S, and X=40, S=30, P=4, T=1, r=10%. We could buy the put at $4, and buy the stock at $30 and borrow the present value of the exercise price at $36.36; therefore, we could get immediate cash of $2.36. Then we could wait till maturity, if the put is out of money, then we could make an extra future profit; in other words, it will also be a moneymachine.

The average transaction fee is about $0.65 per contract fee; thus, we could conclude that the majority of the arbitrage will not exist in the real world since transaction cost > money gained from arbitrage.

## 5. Conclusion

This paper examines upper and lowers bound violations, convexity violations, and put-call parity violations in the market. Black-Scholes model and delta hedging strategy are implemented to exploit any arbitrage profit caused by mispricing. Generally, there are still quite a several mispricing and arbitrage opportunities.

The data shows that the average scale of profit in upper bound arbitrage is about $1.31 per share of put and $1.08 per share of call. The average scale of profit in lower bound arbitrage is about $0.31 per share of put and $0.082 per share of call. So, we can sell the calls and buy the corresponding stocks to make a money machine.

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